\documentclass[11pt]{article}

\usepackage[margin=0.9in]{geometry}

\usepackage{graphicx}

\usepackage{gensymb}

\usepackage{multicol}

\usepackage{amsfonts,amsmath,amssymb}

\usepackage{epstopdf}

\usepackage{tabularx,colortbl}

\usepackage[none]{hyphenat}

\usepackage{fancyhdr}

\usepackage{float}

\usepackage{hhline}

\usepackage{graphicx}

\usepackage{eurosans}

\usepackage{caption}

\usepackage{float}

\usepackage{fourier}

\usepackage{array}

\usepackage{makecell}

\usepackage[nottoc,notlot,notlof]{tocbibind}

\usepackage{graphicx}

\usepackage{colortbl,booktabs}

\usepackage{xcolor}

\newenvironment{rcases}

{\left.\begin{aligned}}

{\end{aligned}\right\rbrace}

\usepackage[]{hyperref}

%\renewcommand\theadalign{bc}

%\usepackage{float}

\newcolumntype{P}[1]{>{\centering\arraybackslash}p{#1}}

\newcolumntype{M}[1]{>{\centering\arraybackslash}m{#1}}

\DeclareCaptionType{mycapequ}[][List of equations]

\captionsetup[mycapequ]{labelformat=empty}

\definecolor{gray}{rgb}{0.85,0.85,0.85}

\pagestyle {fancy}

\fancyhead {}

\fancyfoot {}

\fancyhead [L] {\slshape \MakeUppercase{ 6U Cubesat}}

%\fancyhead [R] {\slshape Innocente Andrea Contalbo}

\fancyfoot [C] {\thepage}

\renewcommand{\footrulewidth}{0pt}

\parindent 0ex

\renewcommand{\baselinestretch}{1}

\rhead{\vspace{-0.5cm}\includegraphics[width=0.3\textwidth]{newlogo.eps}}

\begin{document}

\lfoot{AY 2018-19 -- Prof.\ J.\ D.\ Biggs}

\begin{titlepage}

\begin{figure} [H]

\centering

\includegraphics[scale=0.5]{poli.png}

\end{figure}

\begin{center}

\vspace{1cm}

\Large{\textbf{Department of Aerospace Science and Technology}}\\

%\Large{\textbf{Class Laboratory 1}}\\

\vfill

\line(1,0){400}\\[1mm]

\huge{\textbf{ 6U Cubesat\\Detumbling, Slew Manoeuvre and Sun Pointing}}\\ [3mm]

\Large{\textbf{ Spacecraft Attitude Dynamics And Control \\ Final Project}}\\ [1mm]

\line(1,0) {400}\\

\begin{figure} [H]

\centering

\includegraphics[scale=0.4]{cubesat-1.jpg}

\end{figure}

\vfill

Andrea De Vittori 898352\\

%Candidate \#899783\\

%\today\\

\end{center}

\end{titlepage}

\tableofcontents

\thispagestyle {empty}

\clearpage

%\twocolumn

\setcounter{page}{1}

\section{Introduction}

A CubeSat (U-class spacecraft) is a miniaturized satellite made for space purposes and is composed by multiples of $10\times10\times10$ cm cubic units. The mass is no more than 1.33 kilograms per unit,and often characterized by commercial off-the-shelf (COTS) components for their electronics and structure. CubeSats are commonly put in orbit by deployers on the International Space Station, or launched as secondary payloads on a launch vehicle. The intent is to provide affordable access to space for the university science community, Government agencies and commercial groups thanks to a standardized design of the whole structure. Uses typically involve experiments that can be miniaturized or serve purposes such as Earth observation \cite{intro}. Concerning the 6U (adopted in this project) is essentially the same as two 3Us side-by-side, making it twice as wide . Here listed there are the main features of the mission , deepened and developed through the project:

\begin{enumerate}

\item The satellite is set on a GEO orbit whose aim, after the detumbling phase, is to point the Sun.

\item The detumbling relies on a magnetometer and on a gyro for the attitude and dynamics determination.\\ Subsequently for slew and pointing only the magnetometer is allowed to perform the reconstruction of the the attitude matrix.

\item Since ,After several attempts with magneto torquers, the Cubesat is not able to detumble in a reasonable time span,as an exception, 4 cold thrusters are located on the base of the spacecraft itself.

Regarding the control actuation ,for the last part of the mission, the system is fitted by 4 reaction wheels

\end{enumerate}

\section{List of Parameters}

\subsection{List of selected data for the Simulink simulation:}

\begin{table}[H]

\centering

\begin{tabular}{|M{2cm}|M{5cm}|M{6cm}|M{2cm}|}

\hline

\rowcolor{gray}

\textbf{Variable} & \textbf{Description} & \textbf{Value} & \textbf{Unit of measure}\\

\hhline{|=|=|=|=|}

$a\_{geo}$ &initial semi major axis & 42164 & $[km]$\\

\hline

$e\_{geo}$ & initial eccentricity & 0 & $[-]$\\

\hline

$i\_{geo}$ &initial inclination & 0 & $[rad]$\\

\hline

$\omega\_{geo}$ & initial anomaly of perigee & 0 & $[rad]$\\

\hline

$\Omega\_{geo}$ &initial right ascension of the ascending node & 0 & $[rad]$\\

\hline

$\theta\_{geo}$ & initial true anomaly & 0 & $[rad]$\\

\hline

$\mathbf{\omega\_1}$ & initial angular velocity vector for detumbling & $\begin{bmatrix}

0.22 & 0.26 & 0.22

\end{bmatrix}$& $[rad/s]$\\

\hline

$\mathbf{\omega\_2}$ & initial angular velocity vector for Slew & $10^{-5}\begin{bmatrix}

0.1306 & -0.0114 & -0.4640

\end{bmatrix}$& $[rad/s]$\\

\hline

$\mathbf{\omega\_3}$ & initial angular velocity vector for Pointing & $10^{-4}\begin{bmatrix}

0.4788 & -0.2348 & -0.2283

\end{bmatrix}$& $[rad/s]$\\

\hline

$\mathbf{A\_1}$ & initial attitude matrix for detumbling & $\begin{bmatrix}

0.5335 & 0.808 &0.25 \\

-0.808 & 0.3995 & 0.433\\

0.25 & -0.433 & 0.866

\end{bmatrix}$& $[-]$ \\

\hline

$\mathbf{A\_2}$ & initial attitude matrix for Slew & $\begin{bmatrix}

0.2894 & -0.0961 & 0.9524\\

-0.7044 & 0.6523 & 0.2798\\

-0.6481 & -0.7518 & 0.1211

\end{bmatrix}$& $[-]$ \\

\hline

$\mathbf{A\_3}$ & initial attitude matrix for Pointing & $\begin{bmatrix}

-0.8692 & -0.4429 & -0.2199\\

0.4896 & -0.8335 & -0.2560\\

-0.0699 & -0.3302 & 0.9413

\end{bmatrix}$& $[-]$ \\

\hline

$T\_{detumbling}$ & integration time of detumbling & 120 & $[s] $\\

\hline

$T\_{slew}$ & integration time of slew & 200 & $[s] $\\

\hline

$T\_{pointing}$ & integration time of pointing & 86400 & $[s] $\\

\hline

M & mass of the Cubesat & 12 & $[Kg] $\\

\hline

w & width of the Cubesat & 226.3 & $[mm]$\\

\hline

l & length of the Cubesat & 100 & $[mm]$\\

\hline

a & heigth of the Cubesat & 366 & $[mm]$\\

\hline

$\mathbf{I}$ & Inertia matrix & $\begin{bmatrix}

0.218&0&0\\

0&0.166&0\\

0&0&0.082

\end{bmatrix}$& $[Kgm^2]$ \\

\hline

$f\_g$ & Gyro frequency & 262 & $[Hz]$\\

\hline

$\sigma\_n$ & Gyro Noise & 0.15 & $[\degree/ \sqrt{h}]$\\

\hline

$\sigma\_b$ & Gyro Bias & 0.3 & $[\degree/ h]$\\

\hline

$f\_m$ & Magnetometer frequency & 18 & $[Hz]$\\

\hline

$\sigma\_m$ & Magnetometer pointing acc. & 30 & $[arc/ min]$\\

\hline

$m\_{mt}$ & Magnetic moment of the magneto torquer & 1.2 & $[Am^2]$ \\

\hline

$\mathbf{m\_{SC}}$ & parasitic magnetic moment & $\begin{bmatrix}

0.1 & 0.1 & 0.1

\end{bmatrix}$ & $[Am^2]$\\

\hline

$m\_{max}$ & maximum torque of a RW & 1 & $[mNm]$\\

\hline

\end{tabular}

\end{table}

\begin{table}[H]

\centering

\begin{tabular}{|M{2cm}|M{5cm}|M{6cm}|M{2cm}|}

\hline

\rowcolor{gray}

\textbf{Variable} & \textbf{Description} & \textbf{Value} & \textbf{Unit of measure}\\

\hhline{|=|=|=|=|}

$\mathbf{A}$& matrix disposition of the reaction wheels &$\begin{bmatrix}

1&0&0&\frac{1}{\sqrt[]{3}}\\

0&1&0&\frac{1}{\sqrt[]{3}}\\

0&0&1&\frac{1}{\sqrt[]{3}}

\end{bmatrix}$& [-]\\

\hline

$\mathbf{A^\*}$ & pseudo inverse of $\mathbf{A}$ & $\begin{bmatrix}

\frac{5}{6}&-\frac{1}{6}&-\frac{1}{6}\\

-\frac{1}{6}&\frac{5}{6}&-\frac{1}{6}\\

-\frac{1}{6}&-\frac{1}{6}&\frac{5}{6}\\

\frac{1}{2\sqrt[]{3}}&\frac{1}{2\sqrt[]{3}}&\frac{1}{2\sqrt[]{3}}

\end{bmatrix}$& [-]\\

\hline

$RL\_{RW}$ & rate limiter of a RW & 10 & $[mNm/s]$\\

\hline

F & Thrust of CG thruster & 10 & $[mN]$\\

\hline

$t\_{CG\_{rise}}$&rise time for the CG thruster & 10 & $[ms]$\\

\hline

$t\_{CG\_{fall}}$&fall time for the CG thruster & 50 & $[ms]$ \\

\hline

$\mathbf{C}$ & sensor matrix of the Kalman filter & $$\begin{bmatrix}

1&0&0\\

0&1&0\\

0&0&0

\end{bmatrix}$$& $[-]$\\

\hline

$\mathbf{L}$ & observer matrix of the Kalman filter & $$\begin{bmatrix}

1&0&0\\

0&1&0\\

0&0&0

\end{bmatrix}$$& $[-]$\\

\hline

$k\_{DCM}$& tuning parameter for the DCM filter & 0.1& [-]\\

\hline

$k\_1$& prop.coeff of $\omega\_e$ for the ideal control & 0.1 & $[-]$\\

\hline

$k\_2$& prop.coeff related to $\mathbf{A\_e}$ for the ideal control & 0.005 & $[-]$\\

\hline

\end{tabular}

\end{table}

\section{Structure}

The physical dimensions of the spacecraft are depicted by the following image \ref{dimension}:

\begin{figure} [H]

\begin{minipage}[b]{.68 \textwidth}

\includegraphics[width=\textwidth]{Geometry.PNG}

\caption{ 6U Cubesat dimensions specification \cite{6u dimension}}

\label{dimension}

\end{minipage}

\begin{minipage}[b]{.37 \textwidth}

\centering

\includegraphics[width=\textwidth]{box.PNG}

\caption{ 6-Unit cubesat structure

\cite{structure}}

\label{structures}

\end{minipage}

\end{figure}

The ISIS 6-Unit CubeSat \ref{structures} structure is developed as a modular satellite structure based upon the CubeSat standard. The design created by ISIS allows for multiple configurations, giving CubeSat developers maximum flexibility in their design process.The hardware can be mounted directly inside with stacks or on the frame itself. Every device on board is accessible before mounting external hardware surfaces. Here the main features are shown \cite{structure}:

\begin{multicols}{2}

\begin{itemize}

\item cost: \euro{7350}

\item Outside Envelope $ (l\times w\times h) \ 100\times 226.3 \times 366 \ mm $

\item Primary + Secondary Structure Mass 1.1 kg

\item Inside Envelope $(l\times w\times h) \ per \ module (6 \times) \ 96\times 96 \times 89.4 \ mm$

\item Thermal Range (min $-$ max) -40 to +80 \degree C

\end{itemize}

\end{multicols}

\section{ADCS architecture :}

\subsection{On board instrumentation:}

\subsubsection{gyro:}

\begin{minipage}{.4\textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.34]{Gyro.PNG}

\caption{ STIM300

\cite{gyro}}

\label{gyro}

\end{figure}

\end{minipage}

\begin{minipage}{.6 \textwidth}

The gyro works during the detumbling part for the dynamic determination.It measures the angular velocity of the satellite .STIM300 is a small, tactical grade, low weight, high performance non-GPS aided Inertial Measurement Unit (IMU). It contains 3 highly accurate MEMS gyros, 3 high stability accelerometers and 3 inclinometers. The IMU is factory calibrated and compensated over its entire operating temperature range.

\end{minipage}

\begin{multicols}{2}

\begin{itemize}

\item cost: \euro{\ n.d}

\item Update rate: 262Hz

\item Bias: $<$ 0.3\degree/ h\

\item Noise: $ <$ 0.15\degree/$\sqrt[]{h}$

\item Volume: $35 \ cm^3$

\item Mass: 0.55kg

\item nominal Power:1.5 W

\item Thermal (operational): -40 \degree C to +85 \degree C

\item Insensitive to magnetic fields

\end{itemize}

\end{multicols}

\subsubsection{magnetometer:}

\begin{minipage}{.5 \textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.4]{magnetometer.PNG}

\caption{ NSS Magnetometer

\cite{magnetometer}}

\label{magnetometer}

\end{figure}

\end{minipage}

\begin{minipage}{.5 \textwidth}

The sensor provides x, y and z-axes magnetic field component measurements, in the body reference frame.Mounted outside the spacecraft at the end of a rigid boom the NewSpace Systems magnetometer includes low noise, precision processing and analogue-to-digital conversion circuitry. By knowing the local magnetic field of the Earth it is capable of measuring the attitude kinematics of the spacecraft.Furthermore it is useful for the calculation of magnetorquer rods control torque levels.

\end{minipage}\\\\

\begin{multicols}{2}

\begin{itemize}

\item cost: \euro{14790}

\item Measurement range: -60,000 nT to +60,000 nT

\item Update rate: $<$ 18Hz

\item Resolution: $<$ 8 nT

\item Pointing accuracy: 0.5 \degree

\item Dimensions: $96\times 43 \times 17mm$

\item Mass: $<$ 85g

\item Power:$ < $ 750mW

\item Thermal (operational): -25\degree C to +70 \degree C

\item Power supply: +5V DC

\end{itemize}

\end{multicols}

\subsection{Actuators:}

\subsubsection{Magneto torquer:}

\begin{figure} [H]

\centering

\includegraphics[scale=0.8]{Magneto\_torquers.PNG}

\caption{ NCTR-M012 Magnetorquer Rod

\cite{magneto\_torque}}

\label{rod}

\end{figure}

Magnetorquers offer a way of controlling the attitude of a satellite. This can be attained by means of the interaction of the local Earth's magnetic field.

Operating a magnetic alloy rod produces an amplification effect over an air cored magnetorquer. This requires less power, which is critical for CubeSat missions. The rods can enable a mission with increased manoeuvrability and reduced detumble rates.

CubeSat Magnetorquer rods are designed to be run directly from a switched 5 Volt power output from the on-board power control system.\\In the assigned project only 1 Magnetotorquer is provided with these properties \cite{magneto\_torque}:

\begin{multicols}{2}

\begin{itemize}

\item cost: \euro{1750}

\item Magnetic moment: $1.2 Am^2$

\item Residual moment: $<$ $0.002 Am^2$

\item Operating range: -10 \degree C to +50 \degree C

\item Power: 5 Volt

\item Lifetime: $>$ 10 years

\item Dimensions: 94mm x 15 mm x 13 mm

\item Mass: $<$ 50 g

\end{itemize}

\end{multicols}

This particular rod \ref{rod} has some benefits to take into account . First things first it is a

low cost standard product, it guarantees

high moment for low power and is featured by

small size and low mass , crucial for nano-satellites. In addition it has no residual moment.

\subsubsection{Reaction wheels:}

\begin{figure} [H]

\centering

\includegraphics[scale=0.38]{RW.PNG}

\caption{ CubeWheel Medium

\cite{reaction\_w\_medium}}

\label{RW\_medium}

\end{figure}

The CubeWheel Medium is a reaction wheel adopted to control the attitude of nanosatellites. The module contains a brushless DC motor with vacuum-rated bearings, as well as the required drive electronics and speed control algorithms. Specs:

\begin{multicols}{2}

\begin{itemize}

\item cost: \euro{5,400}

\item Max torque: 1.0 mNm

\item Average power consumption: $<$180 mW (@2000 RPM, 8V)

\item Mass: 130 g

\item Operating voltage: 3.3V / battery voltage (6.5V to 16V) \item Dimensions: 46 x 46 x 31.5 mm

\item mountable in 3 axis

\end{itemize}

\end{multicols}

\begin{minipage}{.5\textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.8]{pyramid.PNG}

\caption{ Reaction wheels layout

\cite{reaction\_w}}

\label{RW}

\end{figure}

\end{minipage}

\begin{minipage}{.5\textwidth}

To work properly , during the slew and pointing phase of the mission , the reaction wheels should be at least 3 mounted on the three axis. In case of redundancy a fourth momentum wheel could be added with the following layout \ref{RW}. Whenever one of the 3 rotors fails the fourth still guarantees the controlability.

\end{minipage}

\subsubsection{CG Thrusters}

\begin{figure} [H]

\centering

\includegraphics[scale=0.6]{Vacco.PNG}

\caption{ CubeWheel Medium

\cite{Vacco}}

\label{Vacco}

\end{figure}

The VACCO CubeSat Hybrid ADN

Reaction Control System is a high

performance micro propulsion system (MiPS)

specifically designed for CubeSats. The smart feed system automatically provides closed-loop thrust vector control during delta-v burns. Reliability is ensured through simplicity of design, welded titanium construction and frictionless valve technology.

\begin{multicols}{2}

\begin{itemize}

\item cost: nd

\item Smart, self-contained propulsion system

\item One 100 mN ECAPS ADN thruster

\item Four 10 mN cold gas ACS thruster

\item Mass 1.797 kg

\item Power: $<$ 15 [W] during hot-fire,$<$ 0.055 [W] in

standby mode

\item $I\_sp$ = 60s

\end{itemize}

\end{multicols}

\section{Power,Mass and Volume Budget:}

\begin{figure} [H]

\centering

\includegraphics[scale=0.78]{table.PNG}

\caption{Budget}

\end{figure}

Please notice that in the mass,volume and power budget additional on board devices are not taken into consideration such as: antennas,batteries,solar panels,.... and you name it.

From the data available the evaluation of the inertia matrix referred to the principal axis is straightforward:

\begin{equation}

\mathbf{I}=

\begin{bmatrix}

0.218&0&0\\

0&0.166&0\\

0&0&0.082

\end{bmatrix} \ \ [kgm^2]

\end{equation}

\clearpage

\section{Model description}

\subsection{Assumptions and approximations:}

\subsubsection{Orbital mechanics:}

%\begin{table}[H]

%\centering

%\begin{tabular}{|P{2cm}|P{2cm}|P{2cm}|P{2cm}|P{2cm}|P{2cm}|}

%\hline

% \rowcolor{gray}

%$\begin{matrix}

%a\_{geo}\\ [km]

%\end{matrix} $ & $e\_{geo}$ & $\begin{matrix}

%i\_{geo}\\ [rad]

%\end{matrix}$ & $\begin{matrix}

%\omega\_{geo}\\ [rad]

%\end{matrix}$&

%$\begin{matrix}

%\Omega\_{geo}\\ [rad]

%\end{matrix}$ & $\begin{matrix}

%\theta\_{geo}\\ [rad]

%\end{matrix}$\\

%\hhline{|=|=|=|=|=|=|}

%42164 & 0 & 0 & 0 & 0 & 0\\

%\hline

%\end{tabular}

%\end{table}

\begin{minipage}{.7 \textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.35]{Perturbations.png}

\caption{ importance of orbital perturbations

\cite{perturbations}}

\end{figure}

\end{minipage}

\begin{minipage}{.3 \textwidth}

The satellite under analysis is set on a GEO orbit. For the the orbital mechanics , as to ease the solution process, the motion is modeled without any kind of perturbation due to : J2,SRP,Magnetic disturbance and so on and so forth. This is certainly a coarse approximation, since in reality the orbit is sensitive to all of them.On the other hand there is no presence of air in GEO orbit, so drag forces can be completely discarded. This picture summarizes the all effects said before:

\end{minipage}

\subsubsection{Satellite dynamics and attitude:}

The aim is to study the local dynamics and to develop for this one control algorithms . In addition, aside from the detumbling which occurs in a small time frame, for slew and Sun pointing the perturbations quoted before are all developed with different models in the following section. It is worth noting that the spacecraft is treated as rigid body, \textit{(no vibrations or continuous mechanics involved)}.

\subsubsection{Sun position:}

\begin{minipage}{.5\textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.79]{ecliptic.PNG}

\caption{ Sun position

\cite{ecliptic}}

\end{figure}

\end{minipage}

\begin{minipage}{.5\textwidth}

\begin{equation}

R\_{Sun}= 149600000 \ [Km]

\end{equation}

\begin{equation}

\omega\_{Sun}=\frac{2 \pi}{365.25 \cdot 24 \cdot 3600 }=1.99 \cdot 10^{-7} \ \left[ \frac{rad}{s} \right]

\end{equation}

\end{minipage}\\\\

Since the inertial reference frame is located in the Earth equatorial reference frame, the position of the Sun will change over time , instead the earth will remain fixed. As first approximation the orbit described by the star is circular with constant angular velocity. Actually this is a good model because the eccentricity the Earth orbit is $ e=0.0167$, defining a nearly circular orbit.

\subsubsection{CG thruster:}

Here the main assumption is that the satellite has a constant mass through the detumbling , when the CG thruster operates.A rough approximation the mass flow rate is given by:

\begin{equation}

\dot{m\_p}=\frac{2F}{Ispg\_E}=3.4\cdot10^{-5} \frac{Kg}{s}

\end{equation}

Considering a detumbling time of 90s, the total expelled mass is :

\begin{equation}

m\_{expelled}=90\cdot3.4\cdot10^{-5}=3\cdot10^{-3} \ \ Kg.

\end{equation}

\subsection{Mathematical Model:}

\subsubsection{Reference frame:}

\begin{minipage}{.5\textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.9]{reference\_frame.PNG}

\caption{ Inertial and body reference frame

\cite{reference\_frame}}

\end{figure}

\end{minipage}

\begin{minipage}{.5\textwidth}

There are different ways to define the attitude of a spacecraft. The one selected for this project is through the direct cosine matrices. Each axis of a reference frame is determined by three components of its unit direction vector.

\begin{equation}

\mathbf{A}=\begin{bmatrix}

u\_1 & u\_2 & u\_4 \\

v\_1& v\_2 & v\_3 \\

w\_1 & w\_2 & w\_3 \\

\end{bmatrix}

\end{equation}

This matrix allows to switch from a representation of a vector from one reference to another one in this way:

\begin{equation}

\mathbf{a\_{uvw}} = \mathbf{A} \mathbf{a\_{123}}

\end{equation}

\end{minipage}

\\\\

The transformation does not affect both the magnitude and the relative orientation of the vectors. These constraints are expressed by the following properties :

\begin{equation}

\mathbf{A}\mathbf{A^T}=\mathbf{I} \ \ \ \ \ \ \ \ and \ \ \ \ \ \ \ \ det(\mathbf{A})=1

\end{equation}

In particular the inertial reference frame \textit{(represented by the Earth itself)} and the body one are identified by :\\

\begin{minipage}{.5\textwidth}

\begin{equation}

\mathbf{N}=\begin{bmatrix}

1 & 0 & 0 \\

0& 1 & 0 \\

0 & 0 & 1 \\

\end{bmatrix}

\end{equation}

\end{minipage}

\begin{minipage}{.5\textwidth}

\begin{equation}

\mathbf{B}=\mathbf{A\_{B/N}}\mathbf{N}

\end{equation}

\end{minipage}

The matrix \ $ \mathbf{A\_{B/N}}$ will be evaluated at each time step , based on the relative orientation of the spacecraft with respect to the Earth.

\clearpage

\subsubsection{Euler Equation:}

The system of differential equations outlining the dynamics of the satellite, are the so called Euler equations:

\begin{equation}

\begin{cases}

\dot{\omega\_x}=\displaystyle\frac{I\_y-I\_z}{I\_x}\omega\_y \omega\_z + \displaystyle\frac{M\_x}{I\_x} \\\\

\dot{\omega\_y}=\displaystyle\frac{I\_z-I\_x}{I\_y}\omega\_z \omega\_x + \displaystyle\frac{M\_y}{I\_y} \\\\

\dot{\omega\_z}=\displaystyle\frac{I\_x-I\_y}{I\_z}\omega\_x \omega\_y + \displaystyle\frac{M\_z}{I\_z}

\end{cases}

\end{equation}

\\ Please notice that $ M\_x,M\_y$ and $ M\_z$ comprises both the disturbances and the control actuation

\subsubsection{Attitude:}

The attitude matrix is governed by this system of differential equations:

\begin{equation}

\mathbf{\dot{A}\_{B/N}}=-[w]^vA

\end{equation}

Despite this relation an instrument on board for detecting the attitude matrix is always needed.

The attitude matrix along with the integration looses the properties of an orthonormal matrix. There is an iterative procedure to retrieve these features :

\begin{equation}

\mathbf{A\_{k+1}}=\frac{3}{2}\mathbf{A\_k}-\mathbf{A\_k}\mathbf{A\_k}^T\mathbf{A\_k}\frac{1}{2}

\end{equation}

\subsubsection{Gravity Gradient torque:}

\begin{minipage}{.5\textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=1]{GG.png}

\caption{ Gravity Gradient Torque

\cite{GG}}

\end{figure}

\end{minipage}

\begin{minipage}{.5\textwidth}

The gravity field is not uniform,thus a torque could act on the satellite. This kind of perturbation is relevant for large satellites due to long time action.In its more generic definition the torque yielded by the elementary force $f$ acting on the infinitesimal mass $m$ is:

\begin{equation}

\mathbf{M\_{GG}}=- \int\_b \mathbf{r} \times \frac{Gm\_E}{|r+r\_{sc}|^3}(\mathbf{r\_{sc}}+\mathbf{r})

\label{gg}

\end{equation}

Where $\mathbf{r\_{sc}}$ is the position vector of the center of mass and $r$ the one from the center of mass to any other point of the spacecraft.

\begin{equation}

r\_{sc} >> r

\end{equation}

\end{minipage}\\\\\\

By making few steps \textit{(here not reported)} and evaluating all the terms under the integral sign, the \ref{gg} turns into:

\begin{equation}

\mathbf{M\_{GG}}= \frac{Gm\_E}{r\_{sc}^3}\begin{bmatrix}

(I\_z-I\_y)c\_2c\_3\\

(I\_x-I\_z)c\_1c\_3\\

(I\_y-I\_x)c\_1c\_2

\end{bmatrix}

\end{equation}

$c\_1,c\_2$ and $c\_3$ are the direction cosines of the radial direction in principal axes.

\clearpage

\subsubsection{SRP:}

\begin{figure} [H]

\centering

\includegraphics[scale=1]{SRP.PNG}

\caption{ reflection and absorption

\cite{SRP}}

\end{figure}

Solar radiation, illuminating the surface of a satellite, determines the presence of a pressure, leading to a torque around the center of mass of the satellite. The are 2 sources of this kind of phenomena : the Sun and the Earth. The former has usually a stronger effect and it can be stated to be constant around our planet. The latter instead, resulted by reflection of the Sunlight is strongly dependent on the distance. The following table summarizes the typical values:\\

\begin{minipage}{.6\textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=1.1]{SRP1.PNG}

\caption{ Typical values of SRP

\cite{SRP}}

\end{figure}

\end{minipage}

\begin{minipage}{.4\textwidth}

The pressure of the electromagnetic radiation can be computed as:

\begin{equation}

P=\frac{F\_e}{c}

\end{equation}

$ F\_e$ is the power per unit surface and $ c$ the speed of light.

Part of the incident radiation is reflected part is absorbed .

\end{minipage}\\\\

As a result the exchange of forces are as follows:

\begin{equation}

\mathbf{F}=PA(\hat{\mathbf{S}}\_b\cdot \hat{\mathbf{n}}\_s)[(1-\rho\_s)\hat{\mathbf{S}}\_b\dot +(2\rho\_s(\hat{\mathbf{S}}\_b\cdot \hat{\mathbf{n}}\_s)+\frac{2}{3}\rho\_d )\hat{\mathbf{n}}\_s]

\end{equation}

The knowledge or the attitude matrix $\mathbf{A\_{B/N}}$ makes the calculation of the Sun position in the body frame easy:

\begin{equation}

\hat{\mathbf{S}}\_b=\mathbf{A\_{B/N}}\hat{\mathbf{S}}\_N

\end{equation}

The satellite is shaped like a cuboid, whose principal axes are oriented as the normal vectors of the outer surfaces:

\begin{equation}

\hat{\mathbf{n}}\_{s1}^b=\begin{bmatrix}

1&0&0

\end{bmatrix}

\ \ \ \ \ \ \

\hat{\mathbf{n}}\_{s2}^b=\begin{bmatrix}

0&1&0

\end{bmatrix}

\ \ \ \ \ \ \

\hat{\mathbf{n}}\_{s3}^b=\begin{bmatrix}

0&0&1

\end{bmatrix}

\end{equation}

\begin{equation}

\hat{\mathbf{n}}\_{s4}^b=\begin{bmatrix}

-1&0&0

\end{bmatrix}

\ \ \ \ \ \ \

\hat{\mathbf{n}}\_{s5}^b=\begin{bmatrix}

0&-1&0

\end{bmatrix}

\ \ \ \ \ \ \

\hat{\mathbf{n}}\_{s6}^b=\begin{bmatrix}

0&0&-1

\end{bmatrix}

\end{equation}\

Last but not least the total torque acting on the satellite is now available:

\begin{equation}

\mathbf{T\_{SRP}}=\begin{cases}

\sum\_{i=1}^{n\_{surfaces}} \mathbf{c\_{pi}} \times \mathbf{F\_i} \ \ \ \ \ if \ \ \ \ \ \hat{\mathbf{S}}\_b \cdot \hat{\mathbf{n}}\_{s\_i}^b \ > \ 0 \\

0 \ \ \ \ \ if \ \ \ \ \ \hat{\mathbf{S}}\_b\cdot \hat{\mathbf{n}}\_{s\_i}^b \ < \ 0

\end{cases}

\end{equation}

\subsubsection{Magnetic disturbance:}

First of all before studying parasitic magnetic moment acting on the spacecraft, it is of paramount importance to derive the magnetic dipole model for the Earth in the inertial frame $ \mathbf{b\_N}$:\\

\begin{minipage}{.36\textwidth}

\begin{equation}

\mathbf{b\_N}=\displaystyle\frac{R\_E^3H\_0}{r^3}[3(\hat{\mathbf{m\_E} }\cdot \mathbf{r\_{sc}})\hat{\mathbf{r\_{sc}}}-\hat{\mathbf{m\_E}}]

\end{equation}

\begin{equation}

H\_0=\sqrt[]{(g\_0^0)^2+(g\_1^1)^2+(h\_1^1)^2}

\end{equation}

\end{minipage}

\begin{minipage}{.64\textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.9]{coefficients.PNG}

\caption{ Gaussian coeff. in nT

\cite{coefficients}}

\end{figure}

\end{minipage}\\\\\\

The versor along the dipole axis $\hat{\mathbf{m\_E}}$ and the position one $\hat{\mathbf{r\_{sc}}}$ are:

\begin{equation}

\hat{\mathbf{m\_E}}=\begin{bmatrix}[sin(11.5 \degree)cos(\omega\_Et)& sin(11.5 \degree)sin(\omega\_Et) &cos(11.5 \degree)

\end{bmatrix}

\end{equation}

\begin{equation}

\hat{\mathbf{r\_{sc}}}=\begin{bmatrix}

cos(\omega\_Et) &sin(\omega\_Et) &0

\end{bmatrix}^T

\end{equation}

Then the magnetic field in the body frame $\mathbf{b\_B} $ is:

\begin{equation}

\mathbf{b\_B}=\mathbf{A\_{B/N}}\mathbf{b\_N}

\end{equation}

It can be measured by the magnetometer installed on board.

The interaction between the local magnetic fields on the spacecraft induced by the flow of current generates a torque given by the formula :

\begin{equation}

\mathbf{M}=\mathbf{m\_{sc}} \times \mathbf{b\_B}

\end{equation}

$ \mathbf{m\_{sc}}$ is the residual magnetic induction . The unit of measure of $ \mathbf{m\_{sc}}$ can be stated as $ Am^2$. In order to simulate the presence of $ \mathbf{m\_{sc}}$ it is reasonable to adopt an average constant value , which expresses a worst case scenario:

\begin{equation}

\mathbf{m\_{sc}} =\begin{bmatrix}

0.1&0.1&0.1

\end{bmatrix} \ \ \ [Am^2]

\end{equation}

\subsubsection{Gyro}

The gyroscopes are implemented on board to measure the angular velocities. The components of this device can be depicted by the image shown below:\\\\

\begin{minipage}{.5\textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.53]{Gyro1.PNG}

\caption{ Gyro

\cite{Gyro1}}

\end{figure}

\end{minipage}

\begin{minipage}{.5\textwidth}

The rotor rotates around the $\mathbf{S}$ axis and the support mechanism spins around the axis $\mathbf{O}$. The Euler equation of the system are:

\begin{equation}

\begin{cases}

\omega\_yJ\_z\dot{\theta}-\overbrace{I\_R}^{rotor}\omega\_R(\omega\_z+\dot{\theta})=M\_x\\

I\_R\dot{\omega\_R}-\omega\_xJ\_z\dot{\theta}=M\_y\\

\underbrace{J\_z}\_{rot+arm+sup} \ddot{\theta}+I\_R\omega\_R\omega\_x=M\_z

\end{cases}

\end{equation}

The first 2 equations provide the reaction forces and the third one the angular velocity $\omega\_x$

\end{minipage}\\\\

As a steady state solution $\omega\_x$ has the following value:

\begin{equation}

M\_z=-k\theta -c\dot{\theta} \Longrightarrow

\omega\_x=-\frac{k\bar{\theta}}{I\_R\omega\_R}

\end{equation}

\clearpage

\paragraph{Simple Gyro noise modeling:\\\\}

Every kind of measure is affected by noise , and the gyro copes with this issue as well:

\begin{equation}

\mathbf{\omega\_{i}^m}=\mathbf{\omega\_i+n+b}

\end{equation}

where $ \mathbf{b}$ is the gyro bias:\\

\begin{minipage}{.5\textwidth}

\begin{equation}

\mathbf{n}=\sigma\_n \mathbf{\zeta\_n}

\end{equation}

\end{minipage}

\begin{minipage}{.5\textwidth}

\begin{equation}

\mathbf{b}=\sigma\_b \mathbf{\zeta\_b}

\end{equation}

\end{minipage}\\\\

$\sigma\_n \mathbf{\zeta\_n}$ and $\sigma\_b \mathbf{\zeta\_b}$ are defined as white Gaussian noise with zero mean value and respectively standard deviation $\sigma\_n$ and $\sigma\_b $ .The source of noise for the first one,known as angular random walk (ARW) , is strictly associated with thermo-mechanical noise.Concerning the second one, called RRW ( rate random walk), is related to the electric noise.

\subsubsection{Magnetometer:}

A magnetometer is an instrument that measures the direction and strength of the magnetic field in the body frame.

This device is operated for the attitude determination by knowing at the same time $b\_B$ and $b\_N$, the local magnetic field and the inertial one.It is usually coupled with a Sun or an Earth horizon sensor, but not in this project. The output $\mathbf{A\_{B/N}}$ is :\\\\

\begin{equation}

\mathbf{A\_{B/N}}=\mathbf{A\_{eps}}\mathbf{A\_{B/N}^\*}

\end{equation}

$\mathbf{A\_{eps}}$ is the error matrix and $\mathbf{A\_{B/N}}^\*$ the ideal attitude:

\begin{equation}

\mathbf{A\_{eps}}=\begin{bmatrix}

cos(\epsilon)cos(\epsilon) & cos(\epsilon)sin(\epsilon)sin(\epsilon)+sin(\epsilon)cos(\epsilon)&-cos(\epsilon)sin(\epsilon)cos(\epsilon)+sin(\epsilon)sin(\epsilon)\\

-sin(\epsilon)cos(\epsilon)&-sin(\epsilon)sin(\epsilon)sin(\epsilon)+cos(\epsilon)cos(\epsilon)&sin(\epsilon)sin(\epsilon)cos(\epsilon)+cos(\epsilon)sin(\epsilon)\\

sin(\epsilon)& -cos(\epsilon)sin(\epsilon)& cos(\epsilon)cos(\epsilon)

\end{bmatrix}

\end{equation}

In a similar manner , as seen with the gyroscope , it exists a model for the noise contribution.

\begin{equation}

\epsilon=p\_{acc}\zeta\_m

\end{equation}

\subsubsection{Magneto torquer:}

A magnetic torquer is a satellite system for attitude control, detumbling, and stabilization built from electromagnetic coils. The magnetorquer creates a magnetic dipole that interfaces with an ambient magnetic field, usually Earth's, so that the counter-forces produced provide useful torque.

\begin{figure} [H]

\centering

\includegraphics[scale=0.7]{Magneto\_torquers1.PNG}

\caption{ Magneto torquer operation

\cite{MAgneto1}}

\end{figure}

The magnetic dipole of the spacecraft is proportional to current flowing in it according to this relation:

\begin{equation}

\mathbf{m\_{MT}}= c\mathbf{I\_{MT}}

\end{equation}

Next the torque is calculated:

\begin{equation}

\mathbf{M}=\mathbf{m\_{MT}}\times \mathbf{b\_b}

\end{equation}

\subsubsection{Reaction wheels:}

A reaction wheel is used primarily by spacecrafts for three axes attitude control, which doesn't require rockets or external applicators of torque. They provide a high pointing accuracy and are particularly useful when the spacecraft must be rotated by very small amounts, such as keeping a telescope pointed at a star.\\\\

\begin{minipage}{.5\textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.7]{RW1.PNG}

\caption{ Reaction wheel model

\cite{RW1}}

\end{figure}

\end{minipage}

\begin{minipage}{.5\textwidth}

Consider the Euler equations under the RW action:

\begin{equation}

\begin{cases}

\mathbf{I}\mathbf{\dot{\omega}}+\mathbf{\dot{\omega}}\times\mathbf{I}\mathbf{\omega}+\mathbf{\omega}\times\mathbf{A}\mathbf{h\_R}+\mathbf{A}\dot{\mathbf{h}\_R}=0\\

\mathbf{u\_c}=-\mathbf{\omega}\times\mathbf{Ah\_R}-\mathbf{A\dot{h\_R}}\\

\mathbf{I\_R\dot{\omega}\_R}=\mathbf{\dot{h}\_R}=\mathbf{T\_R}

\end{cases}

\end{equation}

The control law can be extracted by this set of equations:

\begin{equation}

\mathbf{I}\mathbf{\dot{\omega}}+\mathbf{\dot{\omega}}\times\mathbf{I}\mathbf{\omega}=\mathbf{u\_c}

\end{equation}

From any kind of control the requested $\mathbf{u\_c}$ is known and the equation of interest becomes:

\begin{equation}

\mathbf{\dot{h}\_r}=-\mathbf{A^\*}(\mathbf{u\_{c}}+\mathbf{\omega}\times \mathbf{A}\mathbf{h\_r})

\end{equation}

\end{minipage}\\\\

In particular $\mathbf{A^\*}$ is :

\begin{equation}

\mathbf{A^\*}=\mathbf{A^T}(\mathbf{A}\mathbf{A^T})^{-1}

\end{equation}

\subsubsection{ CG thrusters}

\begin{minipage}{.5\textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.7]{disposition\_thrusters.PNG}

\caption{ disposition of the thrusters

\cite{RW1}}

\end{figure}

\end{minipage}

\begin{minipage}{.5\textwidth}

The easiest way to provide a torque is to generate a set of forces not aligned with the center of mass.For Attitude control cold gas thrusters are very common and capable ,with few milli$-$Newton of applied force, to do the job.The key point is to switch them on and off on cue. They use a non reactive gas (He or N),stored at high pressure (approximately 30 MPa) .

\end{minipage}

\\\\

Given the thruster configuration in the figure, it corresponds a torque matrix computed by taking the cross product between the force of each thruster and the moment arm:\\

\begin{minipage}{.5 \textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=1]{Thrust\_matr.PNG}

\end{figure}

\end{minipage}

\begin{minipage}{.5 \textwidth}

\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ The relative configuration matrix is :

\begin{equation}

\mathbf{T}=

\begin{bmatrix}

\mathbf{T\_1} & \mathbf{T\_2} & \mathbf{T\_3} & \mathbf{T\_4}

\end{bmatrix}

\end{equation}

\end{minipage}

\clearpage

In more detail:

\begin{figure} [H]

\centering

\includegraphics[scale=0.9]{TT\_matr.PNG}

\end{figure}

\section{Control and determination Algorithms}

\subsection{Detumbling}

\begin{figure} [H]

\centering

\includegraphics[scale=0.66]{detumbling.PNG}

\caption{ detumbling}

\end{figure}

\subsubsection{Kalman Filter:}

Kalman filtering is an algorithm that uses a series of measurements observed over time, containing statistical noise and other inaccuracies, and produces estimates of unknown variables that tend to be more accurate than those based on a single measurement alone.It is useful to reconstruct the states of an observable dynamic system:\\\\

\begin{minipage}{.5\textwidth}

\ \ \ \ \ Real system:\\

\begin{equation}

\begin{cases}

\mathbf{\dot{x}}=\mathbf{f(x,u)+\overbrace{w}^{noise}}\\

\mathbf{\underbrace{\mathbf{y}}\_{noisy\ signal}}=\mathbf{g(x,u)+\underbrace{v}\_{noise}}

\end{cases}

\end{equation}

\end{minipage}

\begin{minipage}{.5\textwidth}

\ \ \ \ reconstructed non linear system with noise filtering:\\

\begin{equation}

\begin{cases}

\mathbf{\dot{\hat{x}}}=\mathbf{f(\hat{x},u)+L(y-\hat{y})}\\

\mathbf{\underbrace{ \mathbf{\hat{y}}}\_{filtered \ signal}}=\mathbf{g(\hat{x})}

\end{cases}

\end{equation}

\end{minipage}\\\\

In the assigned problem the reconstructed state is done by selecting $ \mathbf{C}$ as the identity matrix and $ \mathbf{L}$ arbitrarily:

\begin{equation}

\begin{cases}

\dot{\omega}\_{x\_{obs}}=\displaystyle\frac{I\_y-I\_z}{I\_x}\omega\_{y\_{obs}}\omega\_{z\_{obs}}+\mathbf{L\_{1,1}}(\omega\_{x\_m}-\mathbf{C\_{1,1}}\omega\_{x\_m})+\displaystyle\frac{M\_x}{I\_x}\\\\

\dot{\omega}\_{y\_{obs}}=\displaystyle\frac{I\_z-I\_x}{I\_y}\omega\_{z\_{obs}}\omega\_{x\_{obs}}+\mathbf{L\_{2,2}}(\omega\_{y\_m}-\mathbf{C\_{2,2}}\omega\_{y\_m})+\displaystyle\frac{M\_y}{I\_y}\\\\

\dot{\omega}\_{z\_{obs}}=\displaystyle\frac{I\_x-I\_y}{I\_z}\omega\_{x\_{obs}}\omega\_{y\_{obs}}+\mathbf{L\_{3,3}}(\omega\_{z\_m}-\mathbf{C\_{3,3}}\omega\_{z\_m})+\displaystyle\frac{M\_z}{I\_z}

\end{cases}

\end{equation}

\subsubsection{Kalman Filter for DCM}

The attitude matrix is reconstructed in this way:

\begin{equation}

\mathbf{\dot{\hat{A}}}=[\omega+\alpha]^\wedge\mathbf{\hat{A}}

\end{equation}

Where :

\begin{equation}

\alpha=k\_{DCM}(a\_b \times \hat{A}a\_n)

\end{equation}

\subsubsection{Control law for the Magneto Torquer:}

From the theory a simple control law is derived for the magnetic torquers:

\begin{equation}

\mathbf{m}=-m\_{max}sign(\dot{\mathbf{b}}) \ \ \ with \ 1\ magnetic \ torquer \ \ \ \mathbf{m}=-\begin{bmatrix}

m\_{max}sign(\dot{\mathbf{b}\_1})&0 &0

\end{bmatrix}^T

\end{equation}

For a detumbling spacecraft $\dot{b}$ is

\begin{equation}

\mathbf{\dot{b}}=-[\mathbf{\omega}]^v\mathbf{b}

\end{equation}

\subsubsection{Bang Bang Control:}

Knowing the thruster configuration, it can be calculated the maximum torque delivered in each axis $\mathbf{T\_{max\_i}}$. For example :

\begin{equation}

|\mathbf{T\_{max\_1}}|=2lFsin(\alpha)

\end{equation}

As a consequence the maximum torque matrix is:

\begin{equation}

\mathbf{T\_{max}} = \begin{bmatrix}

|\mathbf{T\_{max\_1}}|& 0 &0\\

0& |\mathbf{T\_{max\_2}}| &0 \\

0& 0 & |\mathbf{T\_{max\_3}}|

\end{bmatrix}

\end{equation}

The Bang-Bang control requires the following definition:

\begin{equation}

\mathbf{u}=-\mathbf{T\_{max}}sign(\mathbf{\omega})

\end{equation}

This formula is valid on with 12 or more thrusters , with only 4 thruster it is impossible to have the maximum torque simultaneously on the three axes. As matter of fact a suboptimal solution is found. A logic could be to activate the thrusters , which operate on the axis with the largest angular velocity.\\ For example :

\begin{equation}

\omega\_{max}=\omega\_x \Longrightarrow \mathbf{u}=-\begin{bmatrix}

|\mathbf{T\_{max\_1}}|& 0 &0\\

0& 0&0 \\

0 & 0 & 0

\end{bmatrix}

sign(\mathbf{\omega})

\end{equation}

Also possible rise and fall times and delays of the actuator are taken into account by using the blocks in \textbf {Simulink Rate Limiter and transport delay} .

To avoid chattering the actuation should be switched off when:

\begin{equation}

|\omega\_{max}|< \epsilon\_{CG}

\end{equation}

\subsection{Slew Manoeuvre:}

\begin{figure} [H]

\centering

\includegraphics[scale=0.65]{slew.PNG}

\caption{ Slew Manoeuvre}

\end{figure}

\subsubsection{Sun Pointing:} \label{sun}

The main goal of the Cubesat is to rotate along its axis for an alignment with the Sun, that as first approximation has a fixed position in the space during the slew manoeuvre.Therefore to satisfy this requirement a desired attitude matrix $ \mathbf{A\_d} $ is built in this way :

\begin{equation}

\mathbf{r\_{sc}}=r\_{sc}\begin{bmatrix}

cos(n\_et)&sin(n\_et)&0

\end{bmatrix}

\end{equation}

\begin{equation}

\mathbf{R\_{sun}}=-R\_{sun}\begin{bmatrix}

cos(n\_{sun}t+B)& sin(n\_{sun}t+B)cos(23.45)& sin(n+B)sin(23.45)

\end{bmatrix}

\end{equation}

\begin{minipage}{.4\textwidth}

\begin{equation}

\mathbf{x\_1}=\frac{\mathbf{r\_{sc}}(t^\*)-\mathbf{R\_{sun}}(t^\*)}{|\mathbf{r\_{sc}}(t^\*)-\mathbf{R\_{sun}}(t^\*)|}

\end{equation}

\begin{equation}

\mathbf{x\_2}=\mathbf{x\_3}\times\mathbf{x\_2}

\end{equation}

\begin{equation}

\mathbf{x\_3}=\frac{\mathbf{v\_{sc}}(t^\*)-\mathbf{V\_{sun}}(t^\*)}{|\mathbf{v\_{sc}}(t^\*)-\mathbf{V\_{sun}}(t^\*)|}

\end{equation}

\end{minipage}

\begin{minipage}{.5\textwidth}

\begin{equation}

\mathbf{A\_d}=\begin{bmatrix}

\mathbf{x\_1}\\

\mathbf{x\_2}\\

\mathbf{x\_3}

\end{bmatrix}

\end{equation}

\end{minipage}\\\\

$ \mathbf{x\_1} $is chosen as pointing axis because related to the direction with the maximum inertia, good for stability purposes.

\subsubsection{Ideal Control:}\label{control}

The ideal control tries to figure out which is the correct value of torque that the reaction wheels need to grant.

The expression is :\\\\

\begin{minipage}{.5\textwidth}

\begin{equation}

\mathbf{u\_{c\_{ideal}}}=-k\_1\mathbf{\omega\_e}-\mathbf{e}

\end{equation}

\begin{equation}

\mathbf{\omega\_e}=\mathbf{\omega}-\mathbf{\omega\_d}

\end{equation}

\end{minipage}

\begin{minipage}{.5\textwidth}

\begin{equation}

\mathbf{A\_e}=\mathbf{A}\mathbf{A\_d}

\end{equation}

\begin{equation}

\mathbf{e}=k\_2(\mathbf{A\_e}^T-\mathbf{A\_e})^{V}

\end{equation}

\end{minipage}

\subsubsection{Real Control:}\label{real}

In conclusion the ideal control becomes an input for the reaction wheel dynamics:

\begin{equation}

\mathbf{\dot{h}\_r}=-\mathbf{A^\*}(\mathbf{u\_{c\_{ideal}}}+\mathbf{\omega}\times \mathbf{A}\mathbf{h\_r})

\end{equation}

\begin{equation}

\mathbf{u\_{c\_{real}}}=-\omega\times\mathbf{A^\* h\_r}-\mathbf{A^\*\dot{h\_r}};

\end{equation}

Bear in mind that as for the CG thruster case , a \textbf{rate limiter} is set for the variation of the torque in time granted.

\subsection{Pointing}

\subsubsection{Sun Pointing}

The procedure is the same as seen in \ref{sun}, with a difference: the time is no more fixed so $\mathbf{A\_d}$ will evolve over time.

\subsubsection{Ideal control:}

Here the control law, compared to the Slew case, is transformed in a more complicated formulation because in presence of a time variant attitude. In its more general formulation the relation \textit{(see for further information)} is:

\begin{equation}

\mathbf{u\_{c\_{ideal}}} =-k1\mathbf{\omega\_e}-k\_2(\mathbf{A\_e}^T-\mathbf{A\_e})^{V}+\mathbf{J\omega}\times\mathbf{\omega+J}(\mathbf{A\_e\dot{\omega}\_d-[\omega\_e^{\wedge}] A\_e\omega\_d})

\end{equation}

In the event that the frame evolves slowly in time, a reduced formulation is proposed:

\begin{equation}

\mathbf{u\_{c\_{ideal}}} =-k1\mathbf{\omega\_e}-k\_2(\mathbf{A\_e}^T-\mathbf{A\_e})^{V}+\mathbf{J\omega}\times\mathbf{\omega}

\end{equation}

The following passages for the real control are the same as \ref{real}

\section{Results}\label{results}

\subsection{Detumbling}

\begin{minipage}{0.5\textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.5]{w\_magneto.eps}

\caption{ $\omega\_{detumbling}$ with only magneto torquers}

\end{figure}

\end{minipage}

\begin{minipage}{0.5\textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.6]{w\_det\_cold\_gas.eps}

\caption{ $\omega\_{detumbling}$}

\label{CG}

\end{figure}

\end{minipage}\\\

In a GEO orbit the magnetic field of the Earth is so weak that the time required to detumble a 6U Cubesat is more than 50000 s, making the control actuation useless. This is the reason why CG thrusters are introduced as part of the equipment.

First of all the improvement thanks to the installation of 4 CG thrutsers is noticeable , compared to the previous solution , is able to bring to rest the satellite in less than 100 s. Secondly on the left hand side of \ref{CG} there is the reconstructed $\omega$ and on the right the one measured by the gyro. The Kalman filter combines the ability to follow the input signal with filtering high frequencing oscillation due to noise.

\begin{minipage}{.5 \textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.62]{CGT.eps}

\caption{ CG thruster torque}

\label{CG}

\end{figure}

\end{minipage}

\begin{minipage}{.5 \textwidth}

The \ref{CG} represents the evolution in time of the torque granted by the CG thruster. From this figure it is understandable the functioning of the modified bang-bang control law. The torque is active on the different axes whenever the angular velocity is maximum. Between 40s to 150s, since the various components of $\mathbf{\omega}$ are almost equal in magnitude, there are a lot of firings. It seems that the system is able to generate a variable torque , even if from it is well known that a CG thruster is not throttleable. This is caused by non null ramp and fall times. Furthermore after the de-tumbling the thrusters are not ignited any more in order to avoid chattering .

\end{minipage}

\subsection{Slew Manoeuvre}

\begin{minipage}{.48 \textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.6]{w\_slew.eps}

\caption{ $\omega\_{slew}$}

\end{figure}

\end{minipage}

\begin{minipage}{.52 \textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.68]{A\_slew.eps}

\caption{ $A\_{slew} \ vs \ A\_{desired}$}

\end{figure}

\end{minipage}\\\\

The goal of the Slew is to perform a rest to rest manoeuvre. It has to steer the axes according to the the position of the Sun, which is assumed to be fixed in time in this short time frame. After a transient of 70$-$ 80s it is capable of reaching the desired attitude.\\

\begin{minipage}{.5\textwidth}

\begin{figure} [H]

\centering

\includegraphics[width=\textwidth]{SRP\_slew.eps}

\caption{ $SRP\_{slew}$}

\end{figure}

\end{minipage}

\begin{minipage}{.5\textwidth}

\begin{figure} [H]

\centering

\includegraphics[width=\textwidth]{SPHERE\_SLEW.eps}

\caption{ $3\_d$ motion}

\end{figure}

\end{minipage}\\

A good indicator of the Slew manoeuvre is the SRP perturbation.Indeed if the spacecraft aligns its axes with respect to the sun, only the minimum amount of the outer surface will be illuminated, causing a small contribution of the disturbing torque.\\ Here are reported the other disturbances:\\\\

\begin{minipage}{.5 \textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.62]{GG\_slew.eps}

\caption{ Gravity gradient for slew}

\end{figure}

\end{minipage}

\begin{minipage}{.5 \textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.62]{MDT\_slew.eps}

\caption{ Magnetic disturbance for Slew}

\end{figure}

\end{minipage}

\subsection{Sun Pointing}

\begin{minipage}{.5 \textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.62]{w\_pointing.eps}

\caption{ $\omega\_{pointing} \ for \ 1 \ day$}

\end{figure}

\end{minipage}

\begin{minipage}{.5 \textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale=0.72]{A\_poointing.eps}

\caption{ $A\_{pointing} \ vs \ A\_{desired} \ for \ 1 \ day$}

\end{figure}

\end{minipage}\\\\

Firstly the pointing part is simulated for one day and there are few features to point out:

\begin{enumerate}

\item The spacecraft is almost in a rest configuration because the rate of change of the $ \mathbf{A}$ matrix of the order of 1 day.

\item In addition both $ \mathbf{A}$ and $ \mathbf{A}\_d$ have a period of one day. This can be explained recalling that the orbit is a GEO with 24 H of revolution period . The relative position of the Sun changes a little bit due to the relative motion.

\end{enumerate}

\begin{minipage}{.5 \textwidth}

\begin{figure} [H]

\centering

\includegraphics[scale= 0.61]{A\_365.eps}

\caption{ $A\_{pointing} \ vs \ A\_{desired} \ for \ 1 \ year$}

\end{figure}

\end{minipage}

\begin{minipage}{.5 \textwidth}

\begin{figure} [H]

\centering

\includegraphics[width=\textwidth]{180.eps}

\caption{ attitude of the spacecraft}

\end{figure}

\end{minipage}\\

Secondly the system is integrated for one year and it is important underline this aspect: it is possible to notice 2 vibration modes, the former ,at a high frequency, is brought on by the motion of satellite with respect to the Earth. The latter, at lower frequency, instead is generated by the relative motion of the Sun in the inertial frame.

\section{Conclusions}

Even if a lot of reasonable assumption are made in this project, the steps may be depicted for the work flow of Cubesat mission. The key point is to merge the requirements of the mission, the equipment available on the market, develop a consistent mathematical a physical model and tune the parameters in a acceptable range trying to get the desired results.Concerning the the output of the Simulink file \ref{results} The control and the actuation is robust against both uncertainties of the measurements and outer disturbances. Furthermore the system is always able to adapt itself to new requested configurations in a short time.

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